Appendix A: Course Placement and Sequences

Increased Rigor of Grade 8 and Algebra I/Mathematics I standards

Success in Algebra I/Mathematics I is crucial to students' overall academic success, their continued interest and engagement in mathematics, and the likelihood of their meeting California's A-G requirements. The CA CCSSM represent a tight progression of skills and knowledge that is inherently rigorous and designed to provide a strong foundation for success in the new, more advanced, Algebra I and Mathematics I courses that will typically be taken by most students in the ninth grade.

Development of these skills and knowledge depends on students being placed in the appropriate courses, with emphasis on the appropriate foundational concepts at the appropriate time, throughout their K–8 sequence and beyond. With the help of diagnostic information that is based upon rich common assessments, placement decisions should be reviewed by a team of stakeholders that includes teachers and instructional leadership (Massachusetts Department of Elementary and Secondary Education [Massachusetts] 2012).

Misplacement is common, with negative consequences for students when they are unable to keep pace with the incremental difficulty of mathematics content; students' weaknesses in key foundational areas that support algebra-readiness frequently translate into substantial difficulty reaching proficiency in higher-level mathematics while in high school (Finkelstein, et al., 2012). At the same time, students need to be appropriately challenged and engaged in order to maintain their interest and skill development in mathematics throughout high school and beyond; some students will take college-level courses (Advanced Placement Calculus, Statistics, or International Baccalaureate) as high school seniors, and the course sequences from earlier grades need to support them too. Therefore, one particular placement consideration, discussed later in this chapter, examines when and under what conditions to accelerate students in their mathematics sequence to reach these advanced courses while in high school.

Challenges around Course Sequencing Involving the Transition to CA CCSSM

The implementation of the CA CCSSM comes with many transitions over the next several years – new instructional approaches, new instructional materials, professional support for teachers, and technology readiness, among others. As well, the transition from existing course sequences to new course sequences will inevitably provide challenges at both the school district and school site level. While the fundamental design of new courses presents its own immediate challenges, so too does the linking between courses to ensure vertical articulation between grades, and even between school systems where, for example, K–8 school districts feed into high school only districts. In the particular case of mathematics, there is a “vocabulary” around the names of mathematics courses that is likely to cause confusion not only for educators, but also for parents. “Algebra 1” is a course that, prior to CA CCSSM, has been taught in 8th grade to an increasing number of students. That same course name will be the default for ninth grade for most students who moving forward will complete the CA CCSSM for grade eight – a course that is more rigorous and more demanding than the earlier versions of “Algebra 1.” Even so, we expect the changes to cause confusion. The single most practical solution is to describe detailed course contents, in addition to course names, as a way of clearing up confusion until "Algebra I" as commonly used, refers to a ninth grade and not an eighth grade course.

A Brief Review of Research on Course Placement and Mathematics

Prior research has shown the importance of mathematics course-taking patterns on student achievement. The studies briefly described below provide some additional context for the tradeoffs that are inherent in deciding how best to organize CA CCSSM course sequences, and place students accordingly. Note that references to "Algebra I" refer to courses that were in place under the 1997 CA standards, prior to the adoption of the CA CCSSM. It is also important to note that the CA CCSSM have rigorous grade eight standards—but the California standards adopted in 1997 did not have grade-level specific standards for grade eight. Over the last decade, there has been a dramatic increase in the number and proportion of grade eight students enrolled in Algebra I in California.[1] Williams et al. (2011) report that, between 2003 and 2009, the percentage of grade eight students taking Algebra I increased from 32 percent to 54 percent. While the increase in grade eight enrollment in Algebra I resulted in greater percentages of grade eight students achieving either *Proficient* or *Advanced* on the Algebra I California Standards Test, it also led to larger numbers of grade eight students achieving *Far Below Basic* or *Below Basic* on the test (Williams et al. 2011). Williams et al. (2011) conclude that the practice of placing all eighth graders into Algebra I, regardless of their preparation, sets up many students to fail. Kurlaender, Reardon, and Jackson (2008) looked at students in San Francisco, Fresno, and Long Beach and found that student's

[1] This increase was not confined to California. Similar increases in grade 8 Algebra I enrollment have occurred across the country (Walston and McCarroll 2010; Stein et al. 2011).

grade point average in seventh grade and course failures in eighth grade were predictive of students' high school completion. These authors also found that the timing of when students take algebra is a strong predictor of students' high school success. In two of the three districts that they analyzed, there was a 30 percentage point difference in graduation rates between students who had completed algebra by the eighth grade and those that had not.

As we would expect, and have known for some time, middle school coursework relates closely to high school coursework. Findings from twenty years ago show that course-taking patterns in middle school are highly predictive of course-taking patterns in high school. Oakes, Gamoran, and Page (1992) stated that the courses students take in junior high school are "scholastically consequential, as the choice predicts later placement in high track classes in senior high school" (p. 574). More recently, Wang & Goldschmidt (2003) concluded that middle school mathematics achievement is related significantly to high school mathematics achievement, and that "mathematics preparedness is vitally important when one enters high school – where courses begin to 'count' and significantly affect postsecondary opportunities" (p. 15). In a study examining the National Education Longitudinal Study, Stevenson, Schiller, and Schneider (1994) found that the level of mathematics that students take in eighth grade is closely related to what they take in high school. They conclude "students who are in an accelerated mathematics sequence beginning in eighth grade are likely to maintain that position in high school" (p. 196).

However, many students who finish middle school are not actually prepared to succeed in a rigorous sequence of college-preparatory mathematics courses in high school (Balfanz, McPartland, & Shaw, 2002). Therefore, it is not surprising that previous research has found that among the high school grades, ninth grade is a key year for students in terms of future academic success. Choi and Shin (2004) examine student transcripts from a large, urban school district in California. The authors found that most students fall off-track for college eligibility in the ninth grade. Similarly, Finkelstein and Fong (2008) found that more than 40 percent of the students did not meet the California State University requirement of completing two semesters of college-preparatory mathematics in the ninth grade. They conclude that students who fall off the college-preparatory track early in high school tend to move further from completing a college-preparatory program as they progress through high school. Neild, Stoner-Eby, and Furstenberg (2008) further conclude that the experience of the ninth-grade year contributes substantially to the probability of dropping out of high school, even after controlling for eighth grade academic performance and pre-high school attitudes and ambitions.

The CA CCSSM Grade 8 standards are of significantly higher rigor than the Algebra 1 course that many students have taken while in 8th grade. The CA CCSSM for grade eight address the foundations of algebra by including content that was previously part of the Algebra I course, such as more in-depth study of linear relationships and equations, a more formal treatment of functions, and the exploration of irrational numbers. For example, by the end of the CA CCSSM for grade eight, students will have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. The CA CCSSM for grade eight also include geometry standards that relate graphing to algebra in a way that was not explored previously. In addition, the statistics presented in the CA CCSSM for grade eight are more sophisticated than those previously included in middle school and connect linear relations with the representation of bivariate data.

The New Algebra I and Mathematics I courses build on the CA CCSSM for Grade 8 and are correspondingly more advanced than the previous courses. Because many of the topics previously included in the former Algebra I course are in the CA CCSSM for grade eight, the new Algebra I and Mathematics I courses typically start in ninth grade with more advanced topics and include more in-depth work with linear functions, exponential functions and relationships, and go beyond the previous high school standards in statistics. Mathematics I builds directly on the continuation of the CA CCSSM in grade eight and provides a seamless transition of content through an integrated curriculum.

Because of the rigor that has been added to the CA CCSSM for grade eight, some recalibration of course sequencing will be needed to insure students are able to master the additional content. Specifically, today's students, who are similar to those who may have previously been able to master an Algebra 1 course in grade eight, may find the new CA CCSSM for grade eight content significantly more difficult. This provides an opportunity to strengthen conceptual understanding by encouraging students – even strong mathematics students - to meet the CA CCSSM grade eight standards while enrolled in grade eight.

Recalibrating the course placement process will require school district personnel, including teachers, counselors and instructional specialists to rethink the information they need for assigning students to courses, particularly in middle school mathematics, where many variations may currently exist in the sequence from grade six to grade eight. During the next several years, as implementation of the CA CCSSM strengthens, so too will steps need to be taken at the school district and school site level to insure that the sequence of courses is guiding students to CA CCSSM mastery by the end of grade 8.

Considerations around Mathematics Course Design and Placement under CA CCSSM

Designing CA CCSSM-aligned mathematics courses in middle school requires careful planning to ensure that all content and practice standards are fully addressed. Some students, in some courses, may move through the standards more quickly than others. As noted, however, getting the pacing right will require implementing new courses and examining how students progress. As noted, entering students into a course pathway who are not adequately prepared can have negative consequences. A recent longitudinal analysis based on California statewide assessment data revealed that California's students that fail the state exam for algebra in grade 8 have a greater chance of repeating the course and failing the exam again in ninth grade compared to their peers who pass the state exam for general mathematics in grade eight (Liang, Heckman, and Abedi 2012). Similarly, Finkelstein et al. (2012) reports that as many of one-third of students in a representative sample of California repeated Algebra between grades seven and 12 (most often from grade eight to grade nine), with most not improving their demonstrated mastery following the repeated course. In essence, under standards prior to the adoption of the more rigorous CA CCSSM, California’s eighth graders who were underprepared for algebra were still underprepared in ninth grade.

In light of these findings, school systems across the nation and in California, are revisiting the criteria they use to determine mathematics placement and in the different weights they assign to each criterion. Most districts typically rely on teacher recommendations and course grades to determine course placement (Bitter and O’Day 2010, p. 6), with standardized mathematics test scores, student/parent preferences, and counselor recommendations also factoring into the decision (Hallinan 2003). As Hallinan (1994) notes, “[s]chools vary in the constellation of factors on which they rely to assign students to tracks and in the weight they attach to each factor” (p. 80). Similarly, Oakes, Muir, and Joseph (2000) note: “Increasingly, school systems do not use fixed criteria to assign students to particular course levels” (p.16). Rather, teacher and counselor placement recommendations include subjective judgments about “students’ personalities, behavior and motivation” in addition to test score performance (p. 16).

Research has also shown discrepancies in the placement of students into “advanced” classes by race/ethnicity and socioeconomic background. While decisions to accelerate are almost always a joint decision between the school and the family, serious efforts must be made to consider solid evidence of student learning in order to avoid unwittingly disadvantaging the opportunities of particular groups of students. Among the considerations is the need to assess near-term mathematics readiness with the students' longer-term prospects for mastering advanced mathematics content. The objective districts should follow is when, and under what circumstances, will reinforcing learning through the grade eight CA CCSSM transfer to greater mathematics understanding throughout high school?

In developing district level policy around course sequences and student placement, districts may also turn to guidance from other education agencies. For example, the Achieve Pathways Group has developed a set of clear guidelines on how placement decisions and course sequences should be evaluated based on work published by the Washington Office of the Superintendent of Public Schooling:

> *1. Decisions to accelerate students into the Common Core State Standards for higher mathematics before ninth grade should not be rushed.*
>
> Placing students into an accelerated pathway too early should be avoided at all costs. It is not recommended to compact the standards before grade seven to ensure that students are developmentally ready for accelerated content. In this document, compaction begins in seventh grade for both the traditional and integrated sequences.
>
> *2. Decisions to accelerate students into higher mathematics before ninth grade must require solid evidence of mastery of prerequisite CA CCSSM.*
>
> "Mathematics is by nature hierarchical. Every step is a preparation for the next one. Learning it properly requires thorough grounding at each step and skimming over any topics will only weaken one's ability to tackle more complex material down the road" (Wu 2012). Serious efforts must be made to consider solid

evidence of a student's conceptual understanding, knowledge of procedural skills, fluency, and ability to apply mathematics before moving a student into an accelerated pathway.

3. Compacted courses should include the same Common Core State Standards as the non-compacted courses.

"Learning the mathematics prescribed by CA CCSSM requires that all students, including those most accomplished in mathematics, rise to the challenge by spending the time to learn each topic with diligence and dedication. Skimming over existing materials in order to rush ahead to more advanced topics will no longer be considered good practice" (Wu 2012). When considering accelerated pathways, it is recommended to compact three years of material into two years, rather than compacting two years into one. The rationale is that mathematical concepts are likely to be omitted when trying to squeeze two years of material into one. This is to be avoided, as the standards have been carefully developed to define clear learning progressions through the major mathematical domains. Moreover, the compacted courses should not sacrifice attention to the Standards for Mathematical Practice.

4. A menu of challenging options should be available for students after their third year of mathematics—and all students should be strongly encouraged to take mathematics in all years of high school.

Traditionally, students taking high school mathematics in the eighth grade are expected to take Pre-calculus in their junior years and then Calculus in their senior years. This is a good and worthy goal, but it should not be the only option for students. Advanced courses could also include Statistics, Discrete Mathematics, or Mathematical Decision Making via mathematical modeling. An array of challenging options will keep mathematics relevant for students and give them a new set of tools for their futures in college and career (CCSSI 2010).

Students Who May Be Ready for Acceleration

Understanding that the CA CCSSM are more rigorous than California's previous standards for mathematics, there will still be some students who are able to move through the mathematics quickly. These students may choose to take an accelerated or enhanced mathematics program beginning in eighth grade (or even earlier) so they can take college-level mathematics in high school. However, the previous course sequences for acceleration will need to be updated, considering the increased rigor of the CA CCSSM. Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they are mastering the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students master and fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. There should be a variety of ways and opportunities for students to advance to mathematics courses beyond those included in this publication (CCSSI 2010).

We also note that maintaining motivation and engagement in advanced mathematics is essential for some students who enjoy their work in mathematics and excel in mathematics, and in school, as a result. Slowing down instruction or restricting access to accelerated sequences may discourage and disengage some students from their progress in math, and potentially other courses as well. Therefore, some students may look forward to Advanced Placement (AP) Calculus or Multivariate Calculus as real options for their high school senior year. For high schools that do not offer these courses on a regular basis, concurrent enrollment in local colleges and universities may provide some students an alternative to high school courses.

Districts are encouraged to work with their mathematics leadership, teachers, parents, and curriculum coordinators to design pathways that best meet the needs of their students. Enrichment opportunities should allow students to increase their depth of understanding by developing expertise in the modeling process and applying mathematics to novel and complex contexts. (Massachusetts 2012).

In the CA CCSSM, students begin preparing for algebra in kindergarten, as they start learning about the properties of operations. Furthermore, much of the content central to Algebra I courses of the past—namely linear equations, inequalities, and functions—is now found in the grade eight CA CCSSM. Mastery of the algebra content, including attention to the Standards for Mathematical Practice, is fundamental for success in further mathematics and on college entrance examinations. Skipping over material to get students to a particular point in the curriculum will create gaps in the students' mathematical background. In order to accelerate, students must prove that they are proficient in the CA CCSSM for grades K–8 (CCSSI 2010).

It is essential that multiple measures are used to determine a student's readiness for acceleration. Districts should create a system for gathering evidence to determine if a student is prepared for an accelerated pathway. Placement assessments that include constructed responses should be used to determine students' conceptual understanding. The assessments should incorporate performance items that address multiple domains. In addition, the assessments should measure a student's ability to demonstrate the skills included in the Standards for Mathematical Practice. Many schools and districts in California use commercially produced assessments; however others use valid and reliable district-created exams. A portfolio of student work may be collected as evidence of readiness in addition to student grade reports and assessment data from their previous mathematics courses.

One example of a widely available cognitive diagnostic assessment is the Mathematics Diagnostic Testing Program (MDTP), created through the cooperation of faculty in both the California State University (CSU) and University of California (UC) higher education systems. The testing program was developed to provide students and teachers with diagnostic information about student readiness for a broad range of mathematics courses. This information can help students identify specific areas where additional study or review is needed, and can help teachers identify topics and skills that need more attention in courses. The MDTP tests can be administered online, and the results are immediately available after test completion. Therefore, some districts are exploring using the MDTP test results to assist with placement decisions.

Examples of Accelerated Middle School Pathways

Acknowledging the cautions noted above, a middle school acceleration pathway could compact grade seven, grade eight, and Algebra I or Mathematics I in middle school. The term “compacted” means to compress content, which requires a faster pace to complete, as opposed to skipping content. To prepare students for higher mathematics in eighth grade, districts are encouraged to have a well-crafted sequence of compacted courses. The Achieve Pathways Group has provided “compacted” pathways in which the standards from grade seven, grade eight, and the Algebra I or Mathematics I course could be compressed into an accelerated pathway for students in grades seven and eight, allowing students to enter the Geometry (or Mathematics II) course in grade nine. Details of the “compacted” pathway example can be found in the document *Common Core State Standards for Mathematics Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards*, at http://www.corestandards.org/the-standards. (Massachusetts 2012).

Examples of Accelerated High School Pathways

Due to the critical nature of middle school mathematics, districts may choose to offer high school acceleration options instead of, or in addition to, an accelerated pathway that begins in middle school. Some students may not have the necessary preparation to enter a “Compacted Pathway” but may still develop an interest in taking advanced mathematics, such as AP Calculus or AP Statistics in their senior year. Districts are encouraged to work with their mathematics leadership, teachers, and curriculum coordinators to design pathways that best meet the abilities and needs of their students. For students who study the eighth grade standards in grade eight, there are pathways that will lead them to advanced mathematics courses in high school, such as Calculus. In high school, compressed and accelerated pathways may follow a range of models.

Note that the accelerated high school pathways delay decisions about which students to accelerate while still allowing access to advanced mathematics in grade 12 (Massachusetts 2012). (See the course sequence diagrams which follow this narrative.)

1. Students could “double up” by enrolling in the Geometry course during the same year as Algebra I or Algebra II;
2. Allow students in schools with block scheduling to take a mathematics course in both semesters of the same academic year.
3. Offer summer courses that are designed to provide the equivalent experience of a full course in all regards, including attention to the Standards for Mathematical Practice.[2]
4. Create different compaction ratios, including four years of high school content into three years beginning in ninth grade.
5. Create a hybrid Algebra II/Pre-Calculus or Mathematics III/Precalculus course that allows students to go straight to Calculus in 12th grade (see Enhanced Pathway).
6. Standards that focus on a sub-topic such as trigonometry or statistics could be pulled out and taken alongside the traditional or integrated courses so that students would only need to “double up” for one semester; or
7. Standards from Mathematics I, Mathematics II, and Mathematics III courses could be compressed into an accelerated pathway for students for two years, allowing students to enter the Precalculus course in the third year.

[2] As with other methods of accelerating students, enrolling students in summer courses should be handled with care, as the pace of the courses will likely be fast.

A combination of these methods and the suggested compacted sequences in Appendix A of the Common Core State Standards for Mathematics (CCSSI) would allow for the most mathematically-inclined students to take advanced mathematics courses during their high school career.

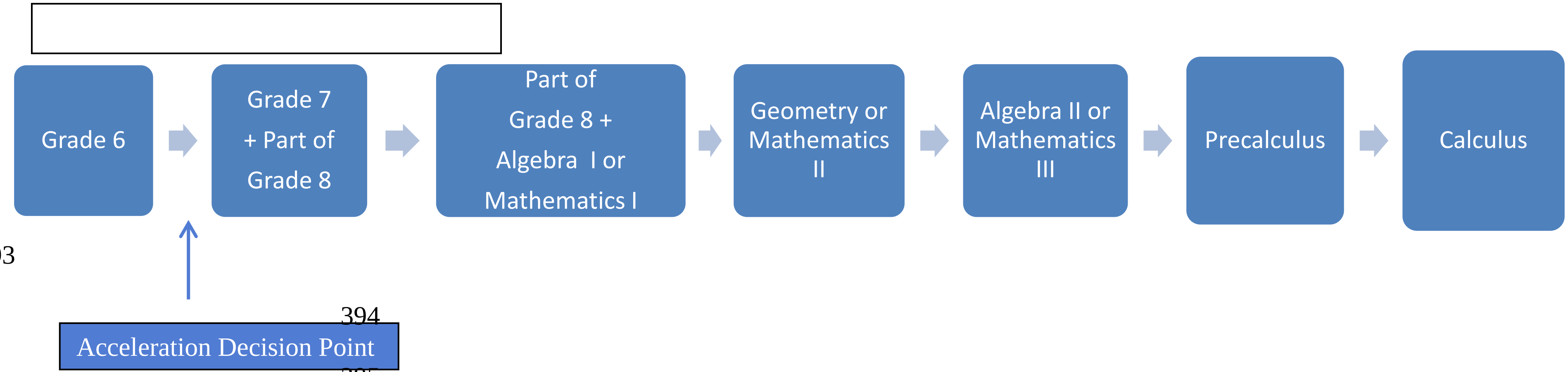
Students Who May Need Additional Support

We expect that students across the state will find the CA CCSSM challenging at all grade levels. For students who have needed additional support to meet existing standards, the CA CCSSM will likely provide still greater teaching and learning challenges. A common existing structural solution in California's public schools has been to encourage students to repeat courses where they have not demonstrated mastery. This has been frequently done between eighth and ninth grade, when concerns about the mastery of pre-algebraic and algebraic content have arisen. Under the CA CCSSM, it is intended that course repetition be reduced for students who need additional support. An alternative is to rethink the content of existing courses in grades six, seven, and eight. Alignment to earlier grades in elementary school will be essential as well to examine how early-grades mathematics standards are being mastered.

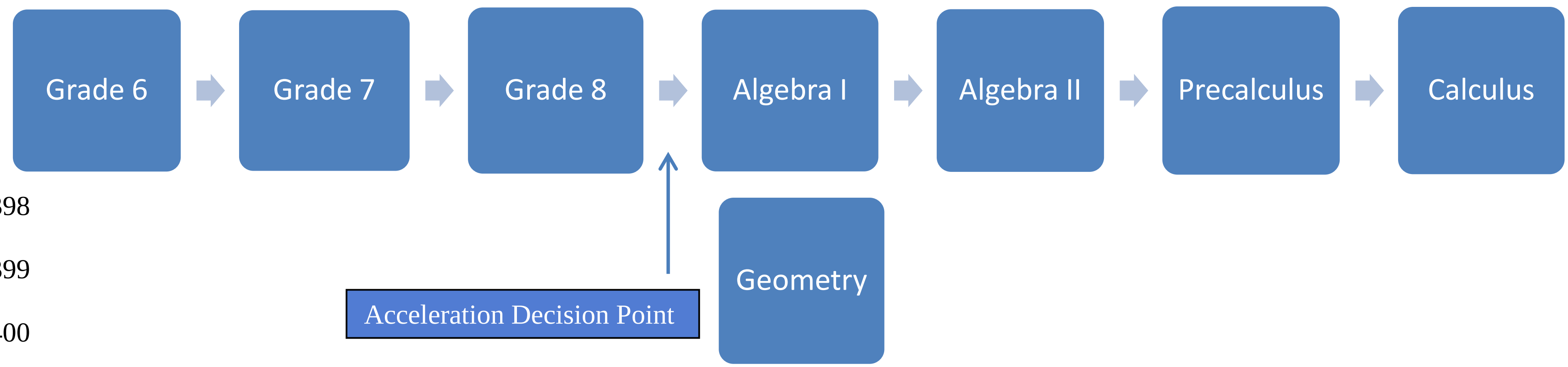
Some districts in California have developed course structures that allow mathematics content to be reinforced over multiple years through *expansion* – the opposite of compaction. Under the CA CCSSM, it is possible that this approach will be helpful, particularly with the assistance of formative testing under the Smarter Balanced Assessment Consortium and other diagnostic testing. Districts should consider how scheduling within the school day, within the school year, and across school years might facilitate increased mastery on the *combined* CA CCSSM from grades six through eight.

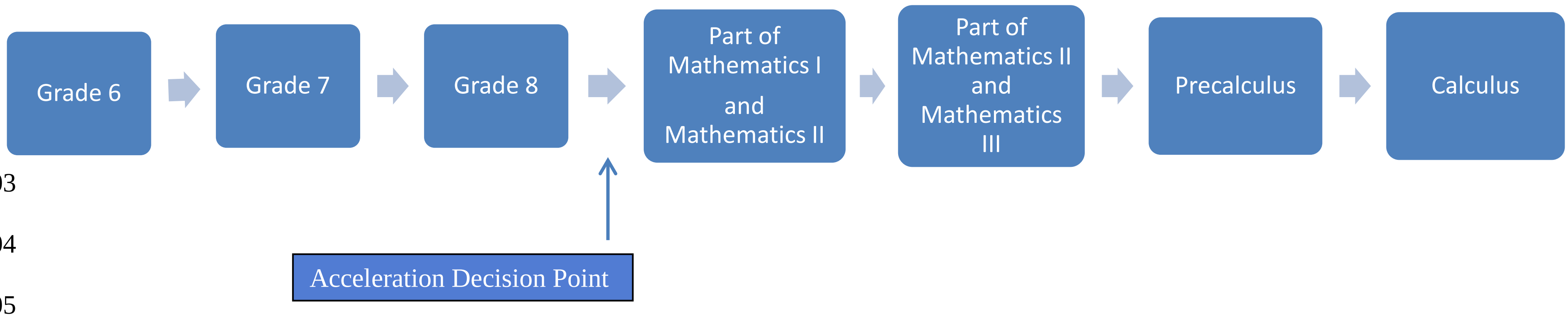
Support for K–12 Teachers

The increased rigor of the CA CCSSM and the demands of fully addressing the MP standards will create additional opportunities and challenges for California's K–12 teachers. Accelerating students who are prepared for advanced coursework will add a new layer to this set of challenges. Students who follow a compacted pathway will be undertaking advanced work at an accelerated pace. This creates a great challenge for these students as well as their teachers, who will be teaching eighth grade standards and Algebra I or Mathematics I standards that are significantly more rigorous than in the past and within a compressed timeframe. Teachers must be prepared not only to address new and more challenging content; but will also need to build upon their repertoire of acceleration strategies. Teacher preparation programs must respond to this call for additional teacher training and support. Support and professional learning for experienced teachers should be provided from the district and county office levels and by the California Mathematics Projects.

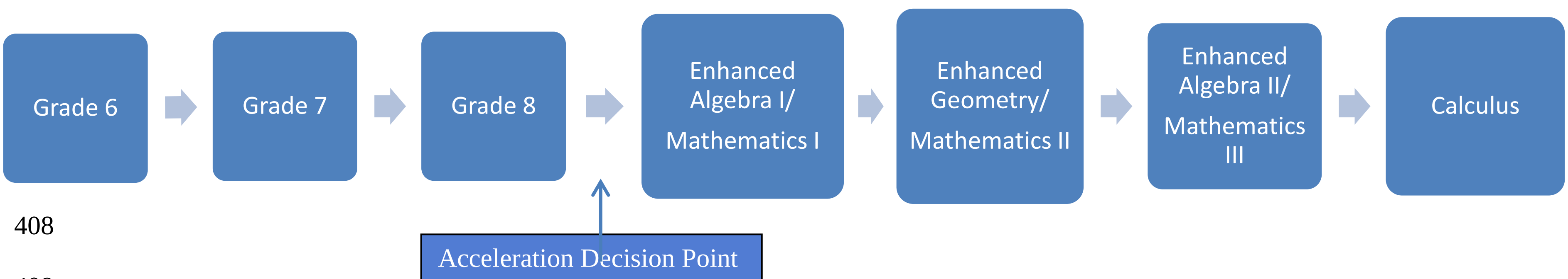


Doubling Up in High School





Enhanced Pathway[3]



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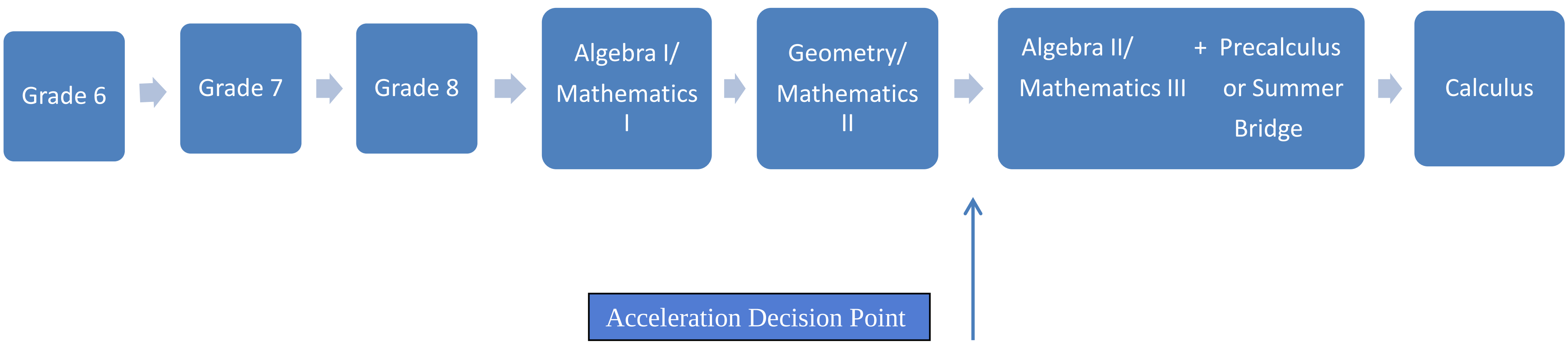
[3] The Massachusetts Department of Education has developed model courses for a traditional enhanced sequence. These are available at: http://www.doe.mass.edu/candi/commoncore/EnhancedPathway.pdf



(Massachusetts 2012)