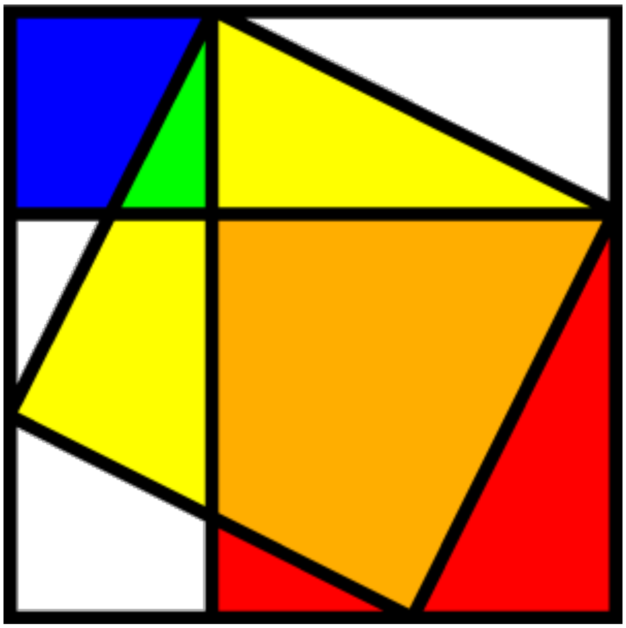
Common Core Mathematics Options:
Assessing Traditional & Integrated Curricular Possibilities



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A Report for the Piedmont Unified School District Mathematics Task Force

by Cheryl Holzmeyer, Ph.D.

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Report Objective and Organization
This report summarizes findings from a research review intended to inform curricular decision-making by the Math Taskforce of the Piedmont Unified School District. In particular, it examines research on "traditional" and "integrated" mathematics curricula, the two approaches possible under California's Common Core State Standards (CCSS). It presents relevant research in an Executive Summary of key findings, followed by an Annotated Bibliography and Discussion of Sources. The Annotated Bibliography includes additional contextual sources and themes not highlighted in the Executive Summary.

Sources
The report draws on a review of articles identified by the Piedmont Unified School District (http://www.piedmont.k12.ca.us/curriculum/common-core/#CCSS_Articles), additional peer-reviewed articles found via JSTOR and ERIC, and a limited spectrum of other sources (grey literature, textbooks, monographs) found via Google and consultations with colleagues.

The research review themes highlighted in the "Annotated Bibliography and Discussion of Sources" (and outlined below), which follows an Executive Summary of key findings, emerged in the course of the research review. These themes are ultimately intertwined in classroom and everyday learning practices.

Annotated Bibliography and Discussion of Sources

1) **Common Core Reforms & Contexts**……………………………………………………5
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Executive Summary

Key Findings:
Below is a summary of findings from the research review, beginning with two caveats to contextualize the points that follow:

1) While the research literature and curricular materials demonstrate differences between traditional and integrated math curricula, including in terms of student learning outcomes, math educational reform debates (or "math wars") that often engage these differences tend to overstate them. In the process, these debates can foster the perception of a stark binary between traditional and integrated curricula that may impair recognition of and engagement with their common elements. This perception may also obscure the potential strengths and weaknesses of each, especially with regards to the preferences and needs of diverse students and teachers, as discussed further below.

2) "Math wars" rhetoric may also inflate the role of curricula alone in students' overall learning experiences, attributing too much causality to curricula apart from the wider array of variables emphasized in the research literature – from teacher professional development to school and community resources. A recent "Best-Evidence Synthesis" (Slavin et al. 2009) of middle and high school math program outcomes found that, "Effect sizes were very small for mathematics curricula," especially compared with daily teaching practices and student interactions.

Integrated &/versus Traditional Mathematics Curricula
That said, the research summarized below suggests that an integrated math curriculum has the potential to facilitate students' mathematical thinking more effectively than a traditional math curriculum – especially in certain areas, such as conceptual understanding. Multiple articles (Grouws et al. 2013; Schoen and Hirsch 2003) indicate this curricular potential.

Perhaps the strongest study of the relative merits of integrated math curricula is by Schoen and Hirsch (2003), in that it examines a broad cross-section of 36 schools and students over a four-year period, as part of the national field test of the Core Plus Mathematics Project (CPMP) integrated math curriculum. It summarizes its core findings as follows:

> "What is most interesting is the consistent pattern of outcomes across the studies. CPMP students almost always performed better than comparison students on measures of conceptual understanding, interpretation of mathematical representations and calculations, and problem-solving in applied contexts, but sometimes not as well on measures of algebraic manipulation skills." (112)

Both these potential strengths of integrated versus traditional math curricula, as well as the potential weaknesses, are important to keep in mind in moving ahead with any curricular adoption and implementation.

Indeed, while the majority of articles highlighted below point to the relative merits of integrated math curricula, it would be inaccurate to portray traditional math curricula, especially in the context of CCSS reforms, as unconcerned with conceptual learning or as necessarily coupled with pedagogies of teacher-controlled "direct instruction" rather than student-centered pedagogies. For example, some advocates of conceptually-oriented traditional curricula (Friedlander and Arcavi 2012) articulate "the potential advantages of the traditional approach" as follows: “Short exercises are readily accessible to students and are easy to implement in regular classrooms, and the focus is on learning one specific skill at a time.” (609) Some teachers and students may find that this approach best supports their success and, crucially, their feelings of success and competence.

The traditional approach also need not be mutually exclusive with an integrated curricular approach. Such binaries tend not to be attuned to the complexity of curricula and the multifaceted dimensions of their implementation. These complexities are evident in the research highlighted below as well as the following curricular diagram, featured in multiple articles (Grouws et al. 2013; Tarr et al. 2013) and developed by the Center for the Study of Mathematics Curriculum (http://www.mathcurriculumcenter.org/), an NSF-funded project by partners at Michigan State University, University of Missouri, Western Michigan University, and the University of Chicago.

“Conceptual Framework For Curriculum Research” (from Tarr et al. 2013):

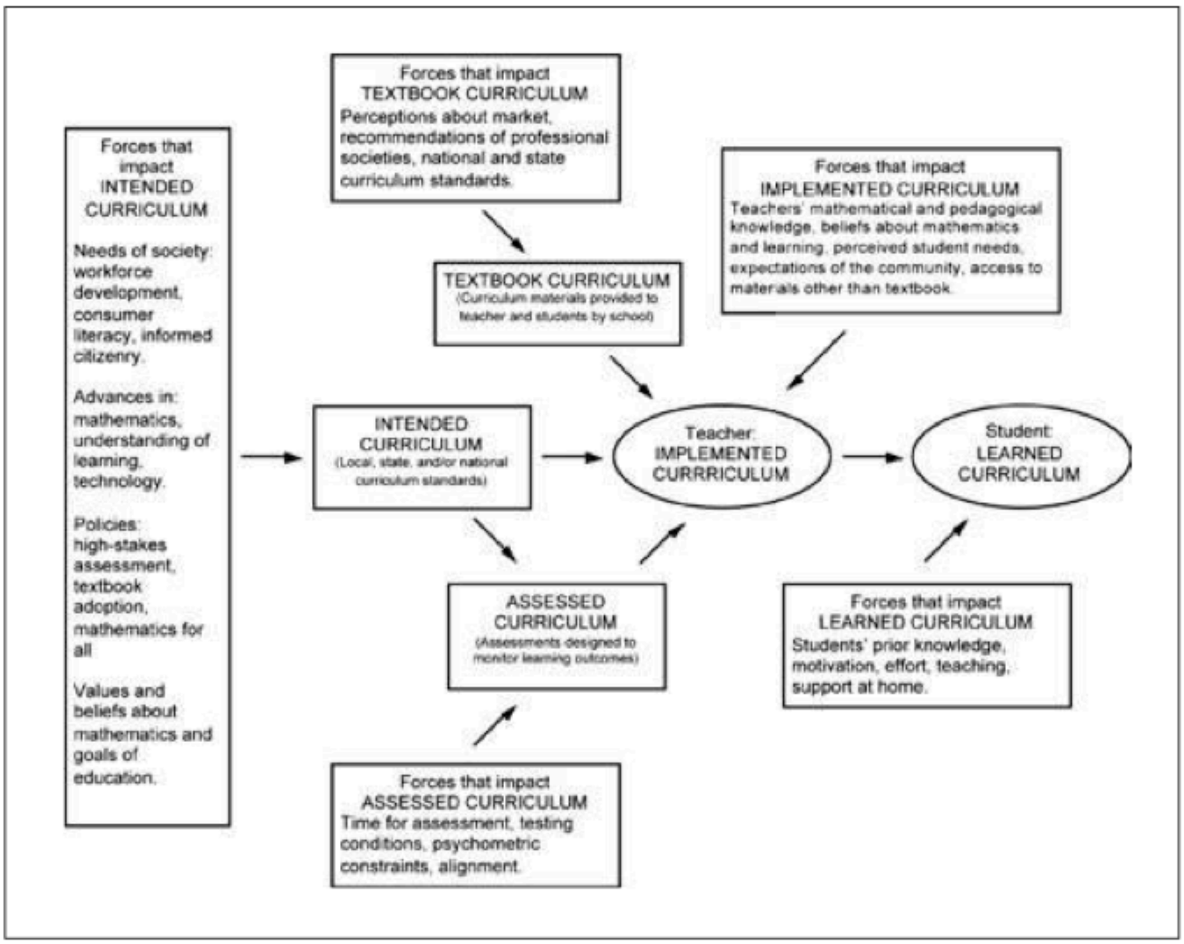


Figure 1. Curriculum Research Framework (Center for the Study of Mathematics Curriculum, n.d.).

Schools, Teachers, and Instructional Practices
Again, multiple articles underscore the importance of thinking systemically, as well as systematically, about how social and classroom contexts shape whatever effects different curricula may bring about. In classrooms, research indicates that relevant dimensions to consider include teachers' own mathematics learning experiences and "narrative identities"; teachers' interpretations of curricular "integration"; teacher professional development, particularly in times of curricular reform; the role and significance of testing in classroom practices (e.g. do teachers feel compelled to "teach to the test"?); and overall classroom conditions and resources (e.g. class size, teacher workload). Curricular change matters most alongside attention to instructional practices and teaching conditions, as highlighted in *Horace's Compromise* (Sizer 2004). In particular, some articles on integrated curricular reform raised the issue of existing standardized tests that may not serve curricular intentions.

"Integrated" Math: Multiple Interpretations and Approaches
One key finding to come out of this research review is the ambiguity of the term "integrated" and its variable interpretation in classroom practices, even after a district adopts a curricular path in the context of the Common Core State Standards for mathematics education. As the studies below elaborate, teachers may bring very different understandings to their engagement with a "single" integrated math curriculum, leading to distinctive instructional practices and student experiences. For example, the contrast between *intra-disciplinary* and *interdisciplinary* approaches to integrated mathematics may be especially crucial to discuss. The distinction between academic and vocational interdisciplinary integration could also be useful to examine.

Explicit discussion of these distinctions by teachers, administrators, students, parents, and other interested community members could help not only to inform curriculum adoption, but also curricular implementation, evaluation, and the ongoing professional development of teachers. It could contribute to thoughtful incorporation and coordination among multiple priorities and multiple approaches to "integrated math" across the district, within the CCSS' overarching framework. This is not to say that a "one size fits all" approach to integrated math, or any other curricula, is desirable or possible – especially given the wide range of students and classroom conditions encountered by teachers. As one article presenting evidence in favor of integrated curricula discusses, "[O]ur findings suggest that carefully thought out adaptations and deviations from textbook content and pedagogical recommendations might be warranted, although we have no data to support wholesale changes to the textbook curriculum." (Tarr et al. 2013) In any case, these variable perspectives are crucial to understanding the complexity of any curricular reforms.

Curricular Materials: Availability?
Whatever CCSS math curricula a school district may formally adopt – whether traditional or integrated – there are a range of views about the current availability of curricular materials (e.g. textbooks) to adequately support and implement the CCSS curricula. This is particularly true for the integrated math curricular pathway, as elaborated below. In addition, the disconnect between high-stakes standardized tests and reform curricula can pose problems for districts trying to adopt and implement such curricula. Some parents, for example, will not want to risk that their children will not do well on standardized exams that affect college admissions or other opportunities. These dynamics are highlighted in this recent article on California school districts' wide array of stances toward traditional versus integrated math curricula: "Districts Split on High

School Math Choices" (http://edsource.org/2014/districts-split-between-choices-for-high-school-math/66169). The article also highlights that opposition to integrated curricula, at least at present, may be due not to any devaluation of its approach, but to the challenges of actually implementing it at this juncture, including the lack of curricular materials. So if the district adopts an integrated math curricular pathway, it will be important to realize that those currently available may not adequately support implementation of integrated math as ideally envisioned. An iterative process of math curricula adoption and implementation may be most appropriate. (Also see **Appendix A:** Pearson Integrated Math Textbook Content Samples, with excerpts from *Mathematics I*, at: http://www.pearsonschool.com/index.cfm?locator=PS262e.)

* * * *

Annotated Bibliography and Discussion of Sources

1) Common Core Reforms & Contexts

Histories of Math Reform and "Math Wars," from Sputnik to the Global Economy

Herrera, T. A. and D. T. Owens (2001) "The 'New New Math'?: Two Reform Movements in Mathematics Education," *Theory into Practice*, Vol. 40, No. 2, Realizing Reform in School Mathematics (Spring 2001), p. 84-92.
This article discusses how opponents of contemporary math education reform contend that it is a reincarnation of the 1960s "new math" movement, often remembered as a pedagogical failure. Yet, as the article elaborates, current National Council of Teachers of Mathematics (NCTM) standards-based reform differs substantially from the new math reform, even while sharing some similarities. In particular, today's reforms are oriented toward all students, not only the college-bound. This principle affects both the content and pedagogy of standards-based reforms, while the emphasis on thinking *processes* is one reason why "[h]igh stakes testing is often not aligned with the *Standards*." (90) This disconnect – between high-stakes standardized tests and reform curricula – can pose problems for districts trying to adopt and implement such curricula.

The following article may be of interest both as a fresh examination of the trajectory of earlier initiatives to reform math curricula, and as material for interdisciplinary classes that seek to place mathematical debates in social and historical contexts:

Philips, C. J. (2014) "In Accordance with a 'More Majestic Order': The New Math and the Nature of Mathematics at Midcentury," *Isis*, Vol. 105, No. 3 (September 2014), p. 540-563.
This article focuses only on the 1960s "new math" reform movement, placing it in the historical context of the Cold War – including the Soviet launch of Sputnik and McCarthyism. "Mid-century educational reformers claimed that more rigorous scientific education would not only solve the claimed 'scientific manpower' shortage but also promote intellectual rigor: academic critics responded to early Cold War exigencies by proposing their own disciplinary methods as models for rational thought generally." (541) As it concludes, "Everyone agreed that students should learn mathematics – and learn it as something more than a stultifying set of facts… Even during a period of nearly unprecedented faith in the benefits of cultivating mathematical ways of thought, the underlying nature of mathematical methods was fiercely contested." (563)

International Comparisons and Perceptions of National Crisis

Many critics of U.S. math education point out that the majority of other countries around the world use math curricula that follow an integrated model of structuring content more than a traditional model. Simultaneously, critics highlight the disparities between U.S. students' math attainments and those of students in a handful of other countries, especially students in Singapore, Japan, and South Korea, among others. International comparisons of U.S. students' mathematics achievements often reference test scores from the Trends in International Mathematics and Science Study (TIMSS) and/or the Program for International Student Assessment (PISA). For example, Evans 2013/2014 reports that the U.S. was ranked 9th out of 48 countries (or administrative regions, such as Hong Kong) in the 2007 TIMSS exam and 9th out of 42 in the 2011 exam, with 5 countries consistently achieving statistically significant higher scores. Critics of U.S. math education may frame these international disparities in math test scores as a national crisis and national security threat, as in the article below (Evans 2013/2014). Looking for answers, they seek to understand how the U.S. could learn from the math educational practices of top-scoring countries, like Singapore, including but not limited to integrated curricular reforms.

However, a wider research review, including the "Best-Evidence Synthesis" cited in the next section (Slavin et al. 2009), suggests that these international comparisons, while important to keep in mind, can magnify the significance of international disparities compared with disparities *within* the U.S. As that review article elaborates, "The problem of mathematics performance in American middle and high schools is not primarily a problem of comparisons to other countries, however, but more a problem within the United States. There are substantial differences between the performance of White and middle-class students and that of minority and disadvantaged students, and the gap is not diminishing…Improvements are needed for all students, of course, but the crisis is in schools serving many poor and minority children." (Slavin et al. 2009: 2)

In addition, focusing principally on international comparisons, especially those that imply integrated curricula may be isolated from other variables affecting students' math educations – including teacher professional development and support – is misleading at best. This point is highlighted in the article below by Fey et al. Thus this research review focuses on comparisons of curricula and other key variables affecting math education in a U.S. context.

* Evan, B. R. (2013/2014) "Mathematics Education in Singapore: How Can Mathematics Education in Singapore Inform Mathematics Education in the U.S," *Mathematics Teaching-Research Journal Online*, Vol. 6, No. 4.
This article's intention is "to inform and improve mathematics education in the U.S. by analyzing literature on Singapore's educational system and exploring aspects that may be helpful to U.S. education." It highlights the TIMSS and PISA assessments as metrics of comparison of student achievement. However, the article acknowledges the limitations on generalizing from Singapore to the U.S.

Fey, J. et al. (2014) "The Future of High School Mathematics," *Mathematics Teacher*, Vol. 107, No 7 (March 2014), p. 488-490.
Article in "Sound Off!" (op-ed) section. Highlights how public outcry over results of the 2012 Program for International Student Assessment (PISA) led to "calls to emulate the curriculum and teaching practices of other countries." And, "[P]olicymakers and pundits with little expertise in mathematics or expertise in mathematics education urged schools to redouble efforts along lines that have been largely ineffective for the past decade and are not common in any high-performing country – a regimen of extensive standardized testing with mostly punitive consequences for schools and for teachers who fail to make adequate yearly progress." (488)

Green, Elizabeth (2014) "Why Do Americans Stick at Math?" *New York Times Magazine*, July 23, 2014
http://www.nytimes.com/2014/07/27/magazine/why-do-americans-stink-at-math.html

2) Studies Comparing Integrated and Traditional Math Curricula

Research Reviews and Meta-Analyses

* Hodara, M. (2013) "Improving Students' College and Math Readiness: Postsecondary Interventions and Reforms," Center for Analysis of Postsecondary Education and Employment (CAPSEE) Working Paper.
This review of research (a working paper) examines "the effectiveness of interventions and reforms that seek to improve the math preparedness and success of high school students entering college." It finds that, while the evidence is limited, short-term programs such as boot camps may in turn have short-term impacts. Also, in terms of interventions "that are strictly pedagogical, the strongest positive evidence is found for using structured forms of student collaboration and for building conceptual understanding through the use of multiple representations when teaching and solving problems." These approaches are supported by CCSS reforms and could be compatible with either integrated or traditional curricula.

* Slavin, R. et al. (2009) "Effective Programs in Middle and High School Mathematics: A Best-Evidence Synthesis," *Review of Educational Research*.
A review article that examined "research on the achievement outcomes of mathematics programs for middle and high schools." It included only studies with either a randomized or matched control group, of at least 12 weeks, and with equality at pretest, of which it found 100, with 26 of these randomized. It found that, "Effect sizes were very small for mathematics curricula and for computer-assisted instruction." It also found that "programs that affect daily teaching practices and student interactions have more promise than those emphasizing textbooks or technology alone." (1) It is important to note that the research review operationalized "mathematics curricula" in terms of textbooks, rather than also including pedagogic dimensions and instructional practices. So its findings support thinking about any curricular reforms – traditional or integrated – in a broader context of classroom practices, student needs, and teacher professional development.

Curricular Comparative Studies and Commentary

Fey, J. et al. (2014) "The Future of High School Mathematics," *Mathematics Teacher*, Vol. 107, No 7 (March 2014), p. 488-490.
Article in "Sound Off!" (op-ed) section. It argues that, "A broad and integrated vision of high school mathematics would serve our students better than the narrow and compartmentalized structure of traditional programs." Then it highlights the need for "balanced attention to technique, understanding, and applications" in math education, coupled with "new and better tools for assessing student learning – especially in the areas of mathematical modeling, problem solving, and quantitative reasoning." Moreover it argues against the potentially detrimental intersection of high-stakes, punitive testing with any reform program: "[W]e need to see that those assessments are used in constructive ways to help teachers improve instruction and to inform educational policy decisions." (489-490) Article also noted above, under International Comparisons.

* "High School Mathematics Pathways: Helping School and Districts Make an Informed Decision About High School Mathematics" (Indiana Department of Education memorandum)
This memo is valuable in highlighting that, under the Common Core, "traditional" curricula are also different than the traditional curricula they are replacing (pre-Common Core). It also addresses, in a cursory way, "Common Misperceptions" of the two curricular pathways; though, these misconceptions are best addressed by the research literature highlighted elsewhere. Finally, it underscores that the state's overarching goal is for students to graduate high school "college and career ready," while acknowledging "this path starts long before students enter high school" – bringing in a broader perspective on the social contexts that intersect with different high school math curricula. (A perspective found in, for example: Shonkoff, J. and D. Philips, eds. (2000) *From Neurons to Neighborhoods: The Science of Early Childhood Development*. National Academy Press: http://www.nap.edu/openbook.php?record_id=9824; also mentioned in last section.)

* Ferguson, K. (2010) "Inquiry-Based Mathematics Instruction Versus Traditional Mathematics Instruction: The Effect on Student Understanding and Comprehension in an Eighth Grade Pre-Algebra Classroom," M.Ed. Thesis, Cedarville University.
Results of this quantitative study of two suburban classrooms, one using traditional math instruction and one using inquiry-based math instruction, "showed that both classes made improvement from their pre-test to their post-test for both units but students receiving instruction through inquiry-based instruction showed significantly more improvement on the second unit." So this study, while limited in temporal scope and numbers of cases (classrooms) studies, suggests the potential of inquiry-based instruction rather than traditional math instruction.

* Schoen, H. and C. Hirsch (2003) "Responding to Calls for Change in High School Mathematics: Implications for Collegiate Mathematics," *The American Mathematical Monthly*, Vol. 110, No. 2.
This study discusses differences that integrated math curricula may make, in terms of student learning outcomes and mathematical skills, compared with traditional curricula, with particular attention to college mathematics preparedness. The article specifically examines the "Contemporary Mathematics in Context: A Unified Approach" or Core-Plus Mathematics Project (CPMP) curriculum (http://wmich.edu/cpmp/) – an integrated, standards-oriented curricula funded in 1992 by the NSF. It examines data from four years of CPMP curriculum development and implementation, including student achievement pre- and post-tests, student attitude surveys, teacher surveys, and structured classroom observations. (110) It mentions that most of these data were part of the CPMP's national field test in 36 schools, representing a broad cross-section of students and schools in urban, suburban, and rural

communities. In terms of outcomes: "What is most interesting is the consistent pattern of outcomes across the studies. CPMP students almost always performed better than comparison students on measures of conceptual understanding, interpretation of mathematical representations and calculations, and problem-solving in applied contexts, but sometimes not as well on measures of algebraic manipulation skills." (112)

The article also discuses the rise of computing technologies, including graphing calculators, in relation to math curricula. For example, it discusses how an early (1983) standards-based reform report by the NCTM suggested the ways that "technology provides an opportunity to devote less time to traditional [manipulative] techniques while boosting understanding and allowing more time for more complex, realistic problem-solving." (109)

* Grouws, D. A et al. (2013) "Curriculum and Implementation Effects on High School Students' Mathematics Learning From Curricula Representing Subject-Specific and Integrated Content Organizations"
This study is informed by the model of the "forces and types of curriculum that influence teacher decision-making and student learning opportunities" – developed by the Center for the Study of Mathematics Curriculum – included in this report at the end of the Executive Summary. With this model as context, the study "examined the effect of 2 types of mathematics content organization on high school students' mathematics learning while taking account of curriculum implementation and student prior achievement. The study involved 2,161 students in 10 schools in 5 states… [S]tudents who studied from the integrated curriculum were significantly advantaged over students who studied from a subject-specific curriculum on 3 end-of-year outcome measures: Test of Common Objectives, Problem Solving and Reasoning Test, and a standardized achievement test. Opportunity to learn and teaching experience were significant moderating factors." (416) In other words, the article offers evidence that an integrated math curriculum is more effective than a traditional curriculum in fostering certain key mathematical practices, including problem solving and reasoning. It simultaneously underscores the importance of thinking systemically, as well as systematically, about how social and classroom contexts shape whatever effects different curricula may bring about. These themes are further highlighted in Section 3.

George, P. S. (1996) "The Integrated Curriculum: A Reality Check," *Middle School Journal*, Vol. 28, No. 1 (Sept. 1996), p. 12-19.
One seasoned teacher's perspective critiquing what he sees as the overselling of integrated curricula. Though for the most part not presenting research evidence himself, he writes, "I have identified at least 36 reasons why we should be cautious, [and] the first 15 relate to unsubstantiated claims about the superiority of integrated curricula." (12) May be useful in exemplifying a highly skeptical perspective on integrated curricula.

Tarr, J., D. Grouws, O. Chavez and V. Soria (2013) "The Effects of Content Organization and Curriculum Implementation on Students' Mathematics Learning in Second-Year High School Courses," *Journal for Research in Mathematics Education*, Vol. 44, No. 4 (July 2013), p. 683-729.
This study examined integrated versus traditional math curricula, based on different content organizational structures, in terms of variable student outcomes at 11 high schools in 5 geographically dispersed states. Its research design was oriented toward examining relationships among curriculum organization, curriculum implementation, and student learning. "Students in the integrated curriculum scored significantly higher than those in the subject-specific curriculum on the standardized achievement test. Significant student-level predictors included prior achievement, gender, and ethnicity. At the teacher level, in addition to Curriculum Type, the Opportunity to Learn and Classroom Learning Environment factors demonstrated significant power in predicting student scores, whereas Implementation Fidelity, Teacher Experience, and Professional Development were not significant predictors." (683) The article also acknowledges that, "the content, focus, and organization of high school mathematics textbooks are likely to undergo significant revisions in the near future," (722) and calls for ongoing studies of curricular efficacy.

Friedlander, A. and A. Arcavi (2012) "Algebraic Skills, A Conceptual Approach: Integrating Procedures and Thinking Processes Makes Learning More Meaningful," *Mathematics Teacher*, Vol. 105, No. 8 (April 2012), p. 609-614.
This article considers a more conceptually-oriented approach to teaching algebra in a traditional curriculum, based "on the potential advantages of the traditional approach: Short exercises are readily accessible to students and are easy to implement in regular classrooms, and the focus is on learning one specific skill at a time." (609) Its approach highlights "constructing examples and counter-examples," "considering and justifying multiple-choice tasks," "meaningful application of algebraic concepts," "qualitative thinking," and "divergent thinking." (611-613) Thus it

emphasizes many of the same types of mathematical thinking that some claim are the exclusive purview of integrated math curricula, and is notable for this reason.

3) Classroom Practices & Curricular Materials

Teachers and Instructional Practices

* McCaffrey, D. et al. (2001) "Interactions among Instructional Practices, Curriculum, and Student Achievement: The Case of Standards-Based High School Mathematics," *Journal for Research in Mathematics Education*, Vol. 32, No. 5, (Nov. 2001).
This article presents "the results of a study investigating the degree to which teachers' use of instructional practices aligned with these reforms is related to improved student achievement, after controlling for student background characteristics and prior achievement." The study examined 10th grade students during the 1997-98 academic year, with some students enrolled in integrated math consistent with NCTM standards reforms, while other students took the more traditional algebra and geometry sequence. It found that, "Use of standards-based or reform practices was positively related to achievement on both tests for students in integrated math courses, whereas use of reform practices was unrelated to achievement in the more traditional algebra and geometry courses. These results suggest that changes to instructional practices may need to be coupled with changes in curriculum to realize effects on student achievement." So this article provides evidence for how curricula matter alongside instructional practices.

Drake, C. and M. G. Sherin (2006) "Practicing Change: Curriculum Adaptation and Teacher Narrative in the Context of Mathematics Education Reform," *Curriclum Inquiry*, Vol. 36, No. 2 (Summer, 2006), p. 153-187.
This article highlights how teachers' own mathematics learning experiences and "narrative identities" are relevant to the possibilities of math education reform and practices of curriculum implementation. It underscores how professional development is critical and pivotal, particularly in times of curricular reform.

Faulkner, V. (2013) "Why the Common Core Changes Math Instruction," *Phi Delta Kappan*, 59-63.
A guide "to support teachers, administrators, and parents as they make important shifts in language to support implementation of the Common Core and discussion of sound mathematics."

Sizer, T. (2004) *Horace's Compromise: The Dilemma of the American High School*.
First published in 1984, this book is now considered a classic in the field of school reform. This more recent version highlights that, while there have been changes in U.S. classrooms over the past thirty years, "much remains the same: rushed classes, mindless tests, overworked teachers."

"Integrated" Math: Multiple Interpretations and Approaches

* de Araujo, Z. et al. (2013) "Teachers' Conceptions of Integrated Mathematics Curricula," *School Science and Mathematics*, Vol. 113 (6).
This study found that "teachers had varied conceptions of what the term integrated meant in reference to mathematics curricula. These varied conceptions led to the development of the Conceptions of Integrated Mathematics Curricula Framework describing the different conceptions of integrated mathematics held by the teachers. The four conceptions—integration by strands, integration by topics, interdisciplinary integration, and contextual integration— refer to the different ideas teachers connect as well as the time frame over which these connections are emphasized. The results indicate that even when teachers use the same integrated mathematics curriculum, they may have varying conceptions of which ideas they are supposed to connect and how these connections can be emphasized. These varied conceptions of integration among teachers may lead students to experience the same adopted curriculum in very different ways." (285)

This study presents the most explicit typology of "integrated" curricula, based on different teacher interpretations, of any research reviewed for this report. This typology highlights both the different content as well as the different types of connection among content areas that teachers may emphasize in their instruction, as they practice "integrated math" in their particular classrooms. More specifically, this study found that teachers may emphasize:

1) **integration by strands** (within the discipline of mathematics, e.g., among algebra, geometry, statistics, etc., throughout a given course or unit);

2) **integration by topics** (within the discipline of mathematics, within a given problem or lesson; time period over which connections made shorter than for integration by strands),
3) **interdisciplinary integration** (ranging from academic disciplines, such as science or art, to vocational subjects), and
4) **contextual integration** (with real-world problems within a given question or lesson).

While this typology is based on one focus-group and interview-based study and should not be assumed to be generalizable, it is notable that these different types of "integration" are implicit in many of the other studies of "integrated curricula" identified below, informed by the categories of education researchers. For example, while some studies highlight the interdisciplinary integration of mathematics with science, art, or vocational education (#3), others focus on integration within the discipline of mathematics (whether along the lines of #1 or #2). Indeed, "integration by strands" (#1) is arguably comparable with the way some teachers approach traditional math curricula, especially as informed by the new Common Core (cf. Friedlander and Arcavi 2012, above). So this typology seems to have at least limited external validity in analyzing a broader, variable sample of approaches to "integrated" math curricula.

Math Intradisciplinary Integration (by Strands and Topics):
Jacobs, H. (1994) *Mathematics: A Human Endeavor, 3rd Edition.*
A book focused on problem solving across fields of mathematics, including how to make observations, discover relationships, and solve problems in the context of ordinary experience. More background on the book at: http://dimacs.rutgers.edu/Volumes/schools/paper82/node3.html

Interdisciplinary Integration:
Bickley-Green, C. A. (1995) "Math and Art Curriculum Integration: A Post-Modern Foundation," *Studies in Art Education*, *37*(1), p. 6-18.
May be especially relevant to engagements with geometry, and discussions of implicit and explicit curricular elements. Paper "presents implicit and explicit curricular assumptions about content in two disciplines. It explores childhood development in art and math as a means of showing how implicit assumptions also affect explicit content in curriculum." (6) Call for further research on "how art and geometry are cognate cognitive structures," and "curriculum writers can reform two courses of study by considering the related webs of meaning between the disciplines before creating impenetrable boundaries." (17)

Isaacs, A., P. Wagreich, and M. Gartzman (1997) "The Quest for Integration: School Mathematics and Science," *American Journal of Education*, Vol. 106, No. 1, Reforming the Third R: Changing the School Mathematics Curriculum (Nov. 1997), p. 179-206.
This article focuses on elementary education and presents a case study of the *Math Trailblazers* curriculum, which attempts "to integrate mathematics and science by emphasizing science as a method and by focusing on a small set of variables that are thought to be fundamental." These materials "are examined in a framework that considers the meaning of math and science integration and its possible advantages and disadvantages, and the difficulties in writing and implementing integrated materials." (179) The article is notable for its assertions of curricular determinism, downplaying the role of teachers and practices of classroom instruction, unlike many other articles in this review.

Oppenheimer, F. (1979) "Aesthetics and the Right Answer," *The Humanist* (March/April 1979); http://www.exploratorium.edu/files/about/our_story/history/frank/pdfs/aesthetics.pdf
Article by San Francisco Exploratorium founder and physicist Frank Oppenheimer. It begins, "Students in physics courses spend a large fraction of their efforts in solving problems and finding the 'right answer.' The backs of most textbooks list the right answers for even-numbered problems, and the students feel guilty and stupid if they cannot find the right answers for the odd-numbered ones. In general, physics is considered a 'right answer' subject. Its metaphysical implications are widely ignored along with the creative nature of scientific activity."

Strober, M. (2010) *Interdisciplinary Conversations: Challenging Habits of Thought*, Stanford University Press.
Though focused on post-secondary education, this book may be useful in fostering dialogue about interdisciplinarity and the complexities of "integrated" curricula among teachers, administrators, and others.

Math Integrated with Career and Technical Education (CTE):
Stone, J. R., C. Alfeld, and D. Pearson (2008) “Rigor and Relevance: Enhancing High School Students’ Math Skills through Career and Technical Education,” *American Educational Research Journal*, Vol. 45, No. 3 (Sept. 2008), p. 767-795.
This study examines a classroom intervention to integrate more mathematics into a high school occupational curriculum – including programs in agriculture, auto technology, business and marketing, health, and information technology. It found that “After 1 year of the math-enhanced CTE lessons, students in the experimental classrooms performed equally o technical skills and significantly better than control students on two standardized tests of math ability (TerraNova and ACCUPLACER).” Notably, the intervention analyzed here includes both curricular and pedagogic dimensions, along with intensive teacher professional development.

Contextual Integration & Real World Problems:
Gutstein, E. R. and B. Peterson (2013) *Rethinking Mathematics: Teaching Social Justice By The Numbers*.
A book with ore than 50 articles addressing “how to weave social justice issues throughout the mathematics curriculum, as well as how to integrate mathematics into other curricular areas.” Includes curricular materials such as lesson plans.

Curricular Materials: Availability?

Whatever CCSS math curricula a school district may formally adopt – whether traditional or integrated – there are a range of views about the current availability of curricular materials (e.g. textbooks) to adequately support and implement the CCSS curricula. This is particularly true for the integrated math curricular pathway.

For example, in his recent (Sept. 21, 2014) white paper, “Common Sense About the Common Core,” Alan Schoenfeld, UC-Berkeley Elizabeth and Edward Conner Professor of (Math) Education, states: “What might be called Common Core curricula – widely accessible curricula intended to be consistent with the common core – don’t really exist yet, although publishers are rushing to get them out.” Full paper available at:
http://gse.berkeley.edu/sites/default/files/users/alan-h.-schoenfeld/CommonSenseCommonCore-AHS20140921.pdf

In addition, a 2012 *Education Week* article, “Educators Craft Own Math E-Books for Common Core,” discusses a group of Utah teachers who concluded that adequate integrated math curricular materials did not exist:

> “There was not a textbook out there that we felt reflected the common core,” said Janet M. Sutorius, a math teacher at Juab High School in Nephi, Utah, who is a co-author. “We felt like the textbook companies were just reorganizing the chapters of their old books.”
> http://www.edweek.org/ew/articles/2012/09/26/05utah.h32.html

The teachers responded by developing their own open-source materials for integrated math instruction, now known as the Mathematics Vision Project (http://www.mathematicsvisionproject.org/).

Subsequent examination of a Pearson Integrated Math I textbook (available for preview online) suggested that, indeed, the textbook represented a remixing of conventional curricular materials more than a deeper engagement with the CCSS integrated math pathway. This assessment was suggested in part by the disconnect between the CCSS integrated math vocabulary and concepts at the beginning of the textbook, and the vocabulary and concepts elsewhere in the book, including the book’s math problems. See Appendix A for sample sections of Pearson Integrated Math I textbook (screenshots from online version available for preview).

When asked via email about the disconnect between curricular materials and the CCSS math curricula, Professor Schoenfeld replied:

> “The issue is general, and not simply about any particular publisher. What I mean by ‘real’ CCSS-consistent materials is materials that fully support the mathematical practices, as well as the content. For the most part commercial publishers have re-shuffled the content of their pre-CCSSM texts in order to claim CCSS-compatibility. However, that’s nowhere near good enough if the original materials didn’t support meaningful development of the practices. What I’d want to see is a coherent series of books/materials that provide deep opportunities for engagement in problem solving, producing & critiquing extended chains of reasoning, and mathematical modeling. Not that I’ve looked that closely, but I haven’t seen anything that does that.” (12/3/14)

In a follow-up question about the relative availability of CCSS-consistent materials for the traditional versus integrated math pathways, he elaborated:

> "The NSF-supported ('NCTM standards-based') curricula were close, in that they aligned reasonably well with the goals of the NCTM Standards, which are the general base for CCSSM. They paid more attention to 'processes' than the other commercial curricula, and that translates some into attention for 'practices' today. But, almost none (maybe none) of those were integrated…" (12/4/14)

So if the district adopts an integrated math curricular pathway, it will be important to research materials and realize that those currently available may not adequately support implementation of integrated math as ideally envisioned. An iterative process of math curricula adoption and implementation may be most appropriate. Some of the references in this research review may also be useful as supplementary materials to support integrated curricula, whether in whole or in part, including Moses (2001) mentioned in Section 4.

Fensterwald, J. (2014) "Districts Split on High School Math Choices," *EdSource* (August 12, 2014): http://edsource.org/2014/districts-split-between-choices-for-high-school-math/66169
Article highlights how some parents may oppose integrated reform curricula because they perceive these reforms as a risk to their children's performance on standardized exams that affect college admissions and other opportunities. The article also highlights that opposition to integrated curricula, at least at present, may be due not to any devaluation of its approach, but to the challenges of actually implementing it at this juncture, including the lack of curricular materials.

Students, Parents, and Reform Curricula

Lubienski, S. T. (2002) "Traditional or Problem-Centered Mathematics?: The Choices of Students and Parents in One District," paper presented at the annual meeting of the American Educational Research Association, April, 2002, New Orleans
Overall, survey and enrollment data indicated that many parents and students in the Ames (Iowa) Community School District, after years of experience with Standards-based instruction, held strong, persistent anti-reform beliefs. Study highlights the difficulty of changing a strongly institutionalized mathematics curriculum. Increased understanding of students' and parents' concerns about mathematics instruction can better position reformers, administrators, and teaching to implement mathematics instruction that best meets students' needs.

Pate, E. P., E. Homestead, and K. McGinnis (1994) "Middle School Students' Perceptions of Integrated Curriculum," *Middle School Journal*, Vol. 26, No. 2 (November 1994), p. 21-23.
This study, in contrast to the one below by Star et al., examines 60 students' perceptions in terms of "likes" and "dislikes" of the integrated curriculum, rather than more fine-grained experiential or interpretive shifts in knowledge. It also presents a breakdown of student responses by gender and assessed ability level – whether high-, average-, or low-ability. It finds striking differences along these axes, though with variable patterns and based on sufficiently small sample sizes such that generalizing about these differences is problematic.

Star, J. R., et al. (2008) "What Students Notice as Different Between Reform and Traditional Mathematics Programs," *Journal for Research in Mathematics Education*, Vol. 39, No. 1 (Jan. 2008), p. 9-32.
This article examines Standards-based reform calculus curricula with pre-reform calculus curricula in general rather than contrasting reform traditional versus integrated curricula. It underscores how students' interpretations of their math learning experiences may be diverse and differ from those of curriculum designers or teachers. Hence, just as teachers' different interpretations of curricula and "integration" may lead to variable learning experiences, so do students' variable interpretations lead to disparate "pockets of experience" across classrooms and learners. Moreover, "high-impact differences, like *Content*, were not always related to curriculum type (reform or traditional)." That said, "Students' perceptions aligned moderately well with those of reform curriculum authors, e.g., concerning *Typical Problems*." This analysis is based on the same constructivist learning theories that frame the CCSS and emphasize students' as active participants in their educations.

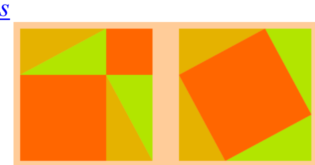
4) Life-Long Learning: Math In and Out-Of-School
Broader Social Contexts, From Schools to Neighborhoods

Martin, L., S. Goldman, and O. Jimenez (2009) "The Tanda: A Practice at the Intersection of Mathematics, Culture, and Financial Goals," *Mind, Culture, and Activity*, 16: 4, p. 338-352.
Presents "an analysis and discussion of the *tanda*, a multiperson pooled credit and savings scheme (a rotating credit association or RCA), as described by two informants from Mexican immigrant communities in California…Contrary to many formal or school-based conceptions of mathematics, mathematical work in the context of the tanda is in service of, and intimately tied up with, cultural goals and values. Likewise, cultural means and mathematics are employed to personal ends. We argue that the tanda should be of enduring interest, particularly among educators interested in bringing more authentic, culturally-relevant mathematics into classroom settings, because it so clearly illustrates how mathematical and cultural processes can interact in the context of personal goals, and provides a potentially valuable template for engaging, consequential, and successful mathematics." This article could be useful for teachers seeking to expand conceptions of out-of-school mathematics and develop lessons to supplement other curricular materials.

Moses, R. P. (2001) *Radical Equations: Civil Rights from Mississippi to the Algebra Project*.
A book about civil rights activist Bob Moses' Algebra Project, begun in 1982 with funding from a MacArthur Genius Grant (http://www.algebra.org/). Its Appendix, "The Mathematics of Trips" – including, "The Constant of Main Street" – seems ideal as one potential curricular element in a larger integrated math curriculum.

Shonkoff, J. and D. Philips, eds. (2000) *From Neurons to Neighborhoods: The Science of Early Childhood Development*. National Academy Press: http://www.nap.edu/openbook.php?record_id=9824
As this book introduces the context of its research synthesis, by a committee established by the National Research Council and Institute of Medicine: "Two profound changes over the past several decades have coincided to produce a dramatically altered landscape for early childhood policy, service delivery, and childrearing in the United States. First, an explosion of research in the neurobiological, behavioral, and social sciences has led to major advances in understanding the conditions that influence whether children get off to a promising or a worrisome start in life. These scientific gains have generated a much deeper appreciation of: (1) the importance of early life experiences, as well as the inseparable and highly interactive influences of genetics and environment, on the development of the brain and the unfolding of human behavior; (2) the central role of early relationships as a source of either support and adaptation or risk and dysfunction; (3) the powerful capabilities, complex emotions, and essential social skills that develop during the earliest years of life, and (4) the capacity to increase the odds of favorable developmental outcomes through planned interventions." (1-2)

Articles below available at: http://www.piedmont.k12.ca.us/curriculum/common-core/
Integrated Approach vs. Traditional Approach
Curriculum and Implementation Effects on High School Students' Mathematics Learning From Curricula Representing Subject-Specific and Integrated Content Organizations – 2013
Responding to Calls for Change in High School Mathematics: Implications for Collegiate Mathematics – 2003
Interactions among Instructional Practices, Curriculum, and Student Achievement: The Case of Standards-Based High School Mathematics – 2001
High School Mathematics Pathways: Helping Schools and Districts Make an Informed Decision about High School Mathematics
Inquiry Based Mathematics Instruction Versus Traditional Mathematics Instruction: The Effect on Student Understanding and Comprehension in an Eighth Grade Pre-Algebra Classroom
Mathematics Education in Singapore: How can Mathematics Education in Singapore inform Mathematics Education in US
Teachers' Conceptions of Integrated Mathematics Curricula
Math Reform
Improving Students' College Math Readiness: Postsecondary Interventions and Reforms
Effective Programs in Middle and High School Mathematics: A Best-Evidence Synthesis



Appendix A: Pearson Integrated Math Textbook Content Samples

Common Core Integrated Math: The red circular graphic beside #27 indicates material deemed especially relevant to the Common Core standards and "Mathematical Practices." The example below illustrates the different degrees of conceptual thinking, interpretation, and problem-solving that may be entailed by these questions, relative to more conventional ones.

26. Travel Suppose a family drives at an average rate of 60 mi/h on the way to visit relatives and then at an average rate of 40 mi/h on the way back. The return trip takes 1 h longer than the trip there.

a. Let d be the distance in miles the family traveled to visit their relatives. How many hours did it take to drive there?

b. In terms of d, how many hours did it take to make the return trip?

c. Write and solve an equation to determine the distance the family drove to see their relatives. What was the average rate for the entire trip?

27. Think About a Plan Each morning, a deli worker has to make several pies and peel a bucket of potatoes. On Monday, it took the worker 2 h to make the pies and an average of 1.5 min to peel each potato. On Tuesday, the worker finished the work in the same amount of time, but it took 2.5 h to make the pies and an average of 1 min to peel each potato. About how many potatoes are in a bucket?

- What quantities do you know and how are they related to each other?
- How can you use the known and unknown quantities to write an equation for this situation?

However, looking at the page that these examples come from (below), these Common Core-type questions are still in the minority. This disparity seemed to be present throughout this textbook.

More Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

B Apply

Solve each equation. If the equation is an identity, write *identity*. If it has no solution, write *no solution*.

18. $3.2 - 4d = 2.3d + 3$
19. $3d + 4 = 2 + 3d - \frac{1}{2}$
20. $2.25(4x - 4) = -2 + 10x + 12$
21. $3a + 1 = -3.6(a - 1)$
22. $\frac{1}{2}h + \frac{1}{3}(h - 6) = \frac{5}{6}h + 2$
23. $0.5b + 4 = 2(b + 2)$
24. $-2(-c - 12) = -2c - 12$
25. $3(m + 1.5) = 1.5(2m + 3)$

26. Travel Suppose a family drives at an average rate of 60 mi/h on the way to visit relatives and then at an average rate of 40 mi/h on the way back. The return trip takes 1 h longer than the trip there.

a. Let d be the distance in miles the family traveled to visit their relatives. How many hours did it take to drive there?

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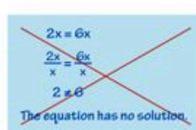
- What quantities do you know and how are they related to each other?
- How can you use the known and unknown quantities to write an equation for this situation?

28. Error Analysis Describe and correct the error in finding the solution of the equation $2x = 6x$.



29. Skiing A skier is trying to decide whether or not to buy a season ski pass. A daily pass costs \$67. A season ski pass costs \$350. The skier would have to rent skis with either pass for \$25 per day. How many days would the skier have to go skiing in order to make the season pass less expensive than daily passes?

30. Health Clubs One health club charges a \$50 sign-up fee and \$65 per month. Another club charges a \$90 sign-up fee and \$45 per month. For what number of months is the cost of the clubs equal?